

Teaching Mathematics Meaningfully

Solutions for Reaching Struggling Learners

Second Edition

by

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Preface

Welcome to the second edition of *Teaching Mathematics Meaningfully: Solutions for Reaching Struggling Learners*. Since the first edition, additional research has been published detailing how students in general, students with disabilities, and other struggling learners progress in their reasoning and understanding in various mathematical domains. Also, with the advent of the Common Core State Standards (CCSS), along with reform movements such as the increased utilization of multi-tiered systems of support (MTSS) and response to intervention (RTI) models, the landscape of mathematics education in schools is changing. The second edition is substantially revised to reflect those new developments.

Although this edition has the same intent as the first—to help teachers of struggling learners teach mathematics meaningfully—it differs substantially in organization and content. We hope readers of the first edition find this one to be a logical and helpful “next step.” For those who have not read the first edition, the second is our best attempt at integrating the current literature related to research and practice in mathematics instruction for struggling learners to provide you valuable insight and information into improving mathematics outcomes for them. This edition incorporates findings from updated research since the first edition, focusing on students’ learning trajectories in mathematics and integrating that information with effective instructional practices for struggling learners within the context of the CCSS and MTSS/RTI. This preface explains the purpose and intended audience of the book, its organization, and how it can be used to further your professional development.

PURPOSE

Meaning is the seminal theme of this book. As a consequence, its purpose is to help teachers facilitate struggling learners’ access to high-quality mathematics so those learners can make sense of the mathematics and become mathematically proficient. To facilitate such access, teachers must be effective problem solvers and decision makers. This book is organized to provide teachers of mathematics and those who want to become teachers of mathematics with an informed and practical process for doing this very important job.

Two primary bodies of literature and practice—special education and mathematics education—inform the book’s content. We represent both disciplines, and for this book to achieve its purpose, collaboration between these two disciplines is essential. Both content and pedagogy must be represented and integrated in an informed and accurate way if K–12 mathematics content is to be made accessible to struggling learners.

AUDIENCE

This book is written for you, wherever you are in your teaching journey, as you strive to enhance your ability to teach mathematics effectively to struggling learners. Special educators, elementary and middle school educators, and prospective and in-service teachers alike can all benefit from this book. A critical component of the instructional framework we present is that educators understand the big ideas of mathematics in general and recognize the value of teaching these ideas. That said, it is assumed that you have at least a basic understanding of the mathematics curriculum—that is, the concepts and skills—that you are responsible for teaching. If not, then it is assumed that you have access to additional resources that will help you develop not only conceptual understanding but also procedural flexibility and fluency for the mathematics included in your curriculum. *The Teaching Student-Centered Mathematics Series* (e.g., van de Walle, Bay-Williams, Lovin, & Karp, 2014; van de Walle, Karp, Lovin, & Bay-Williams, 2014; van de Walle, Lovin, Karp, & Bay-Williams, 2014) includes excellent resource books that can help teachers enhance their mathematical as well as their pedagogical content knowledge. (Note that teaching “student-centered” mathematics means to start with the student’s understanding and needs, using instructional methods based on how children conceive of mathematical ideas versus how adults do and using specific kinds of teaching and learning experiences and feedback that struggling learners need to understand particular mathematical concepts.) Likewise, books such as *Number Sense and Number Nonsense: Understanding the Challenges of Learning Math* (Krasa & Shunkwiler, 2009) can help you better understand the effect of students’ disabilities on their learning of mathematics.

HOW THE BOOK IS ORGANIZED

The book begins with an introductory chapter followed by 11 additional chapters organized in four parts. Chapter 1 introduces the five components of meaningful and effective mathematics instruction for struggling learners that set the conceptual framework for the book. Each component is explored in depth in subsequent chapters; they are integrated within the Teaching Mathematics Meaningfully Process, discussed at length in Chapter 12.

Part I includes Chapters 2–3 and emphasizes the connections between the learner and the mathematics. Chapter 2 introduces the big ideas in mathematics content and the standards of mathematical practice that teachers need to be aware of as they plan their lessons and carry out effective instruction, whereas Chapter 3 provides critical information on four learning trajectories that illustrate how children’s mathematical thinking in number and operations develops over time. Learning trajectories such as these can help deepen your pedagogical content knowledge so you can build more specific and useful models of student thinking, effectively analyze students’ work, determine the relative difficulty of mathematical tasks, productively interact with students during instruction, and identify places where the curriculum needs to be modified to help students progress in their understanding.

Part II, which includes Chapters 4–5, focuses on the needs of struggling learners as well as the necessity of continuous assessment. Chapter 4 describes common mathematics performance traits among struggling learners and how

nine typical learning characteristics and five curriculum factors can create barriers for these students. Chapter 5 describes important formative assessment practices that can be effectively utilized when teaching targeted mathematics concepts and skills to understand what students know, what they don't know, and why. We also provide an overview of the wide range of assessments utilized within MTSS/RTI.

Part III describes how to integrate different components to plan and implement responsive instruction. Part III includes Chapters 6–11, which describe teaching practices that will ultimately inform responsive instruction and support teaching mathematics meaningfully. These chapters emphasize the importance of making informed decisions about your overall instructional approach. In Chapter 6, we introduce ways to think about where to focus your instruction at any point in time across two instructional continua—more teacher directed to less teacher directed, and more explicit to more implicit—based on your students' needs and the characteristics of the mathematics content. Chapter 7 builds on this information by identifying and describing 11 research-supported instructional approaches targeted toward the needs of struggling learners in mathematics. We call these the Essential Instructional Approaches, or EIAs. Chapter 8 introduces the high-leverage Effective Mathematics Teaching Practices (MTPs) promoted by the National Council of Teachers of Mathematics (NCTM; 2014) *Principles to Actions* and their importance for supporting student engagement in rigorous mathematics. This chapter focuses on how the EIAs discussed in Chapter 7 can be used with these MTPs to provide struggling learners access to deeper levels of mathematical understanding—including strategies for promoting initial understandings and proficiency and for generalizing mathematical understanding to multiple contexts.

In Chapter 9, we summarize the current research base related to mathematics instruction and struggling learners and provide a brief introduction to the MTSS/RTI frameworks. MTSS and RTI are discussed further in Chapters 10–11, which focus on how to make effective mathematics instructional decisions individually and school-wide. Chapter 10 describes how to use the practices discussed in Chapters 6–9 to intensify instruction for struggling learners across instructional tiers within MTSS/RTI. Chapter 11 provides decision-making tools that can be used by teachers, grade-level teams, and school-wide teams to evaluate how mathematics instruction is being systematically intensified across tiers.

Finally, in Part IV, Chapter 12, we reiterate and summarize each component of the process for teaching mathematics meaningfully, establishing a decision-making framework that integrates information from the previous 11 chapters. This chapter will help educators systematically and effectively make mathematics instructional decisions for struggling learners within MTSS/RTI. It can be used by you individually, by you and your co-teachers, and by teams involved in problem solving and decision making.

Throughout the book, our approach integrates two disciplines, special education and mathematics education. Recognizing the challenges inherent in integrating these disciplines, and the changes often required of both teachers and learners, we have added Take Action sections at the end of each chapter with activities to encourage readers to reflect as they learn new practices for teaching mathematics to struggling learners. You can find blank photocopiable versions of the forms used in these activities in Appendix A; see also the About the Activities and Forms page for information on how to download printable copies.

Appendix B provides a blank ARC Assessment Planning form, Appendix C provides a peer-tutoring activity from the first edition that we found useful, and Appendix D provides you with an opportunity to see how teachers would use think-alouds in instruction. Finally, Appendix E includes an illustrative case study with reflection opportunities. You can use it to visualize how the Teaching Mathematics Meaningfully Process can be implemented for instructional decision making and to practice contemplating decisions you might make that are similar to or different from those made by the case study teachers.

HOW TO USE THIS BOOK FOR PROFESSIONAL DEVELOPMENT

Whether you are a new or veteran teacher, there is always more to learn about the mathematics you are teaching as well as the most effective methods of teaching diverse learners. Effective teachers consistently strive to improve their practice through various avenues—reading professional articles and books, engaging in productive discussions with colleagues, and reflecting on their successes and opportunities for growth. This book can be used in a variety of settings to encourage professional growth: in school- or division-wide professional development sessions, in a grade-band professional learning community, in small groups of collegial teachers, or within a preservice education program.

In all these settings, reflection and discussion with other instructors are crucial. Guided opportunities for reflection are provided in the Take Action Activities, which are meant to guide your reflection on each chapter's big ideas and suggest ways to put your developing understanding into action. We recommend that regardless of what setting you work in, you consistently and purposefully engage with others in these efforts so that you can support each other as you learn and implement this book's suggested practices. We encourage you to use these questions and activities to expand your thinking about ways to support struggling learners in your classroom.

The case study in Appendix E provides further opportunities to think about how to use the five components of meaningful and effective mathematics instruction for struggling learners. The case study describes the instructional decisions made within each component by a general education mathematics teacher and a special education teacher. As you begin to implement these five components in your own lessons, we encourage you to keep a journal of the successes and challenges you encounter.

Trying to change everything in your teaching at once is unrealistic and potentially overwhelming. Select one or two practices at a time to incorporate. Identify and reflect on their implementation as described previously, looking for ways to capitalize on the successes and mitigate the challenges. Look back through your journal to find patterns in what is consistently working or consistently emerging as a challenge.

Before you begin reading this book, we offer these final thoughts to help you be purposeful in your plans for growth and development:

- *Think about your story and the stories of your students who struggle in mathematics. What drew you to read this book? How are you hoping to improve your ability to support struggling learners in mathematics?*

- *Make a plan for getting the most out of this book.* Reading it cover to cover will provide you with a comprehensive roadmap for reaching struggling learners in mathematics. You can also pick and choose chapters based on your areas of greatest need. Which strategy makes more sense for you?
- *Bring a friend along.* As much as you will benefit from reading this book by yourself, research clearly indicates that you will benefit even more if you read and discuss what you are learning with another colleague (Desimone, 2009). If you are not involved in official professional development sessions or a teacher preparation program, embrace the challenge and ask someone else to join you on this journey. Even if you are involved in these sessions, we strongly recommend that you seek out colleagues with whom to have continued discussions during and between sessions.

FINAL REMARKS

We hope that educators find the organization of the book helpful for furthering their professional development in effectively teaching mathematics for struggling learners. We wish you success on your journey toward achieving improved mathematics learning outcomes for struggling learners in K–12 schools.

REFERENCES

- Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher*, 38(3), 181–199.
- Krasa, N., & Shunkwiler, S. (2009). *Number sense and number nonsense: Understanding the challenges of learning math*. Baltimore, MD: Paul H. Brookes Publishing Co.
- National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematics success for all*. Reston, VA: Author. Retrieved from <http://www.nctm.org>
- van de Walle, J., Bay-Williams, J., Lovin, L., & Karp, K. (2014). *Teaching student-centered mathematics: Grades 6–8* (2nd ed.). New York, NY: Pearson.
- van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Grades 3–5* (2nd ed.). New York, NY: Pearson.
- van de Walle, J., Lovin, L., Karp, K., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Grades preK–2* (2nd ed.). New York, NY: Pearson.

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Critical Components of Meaningful and Effective Mathematics Instruction for Students with Disabilities and Other Struggling Learners

The phrase *struggling learners* can mean different things to different people. In this book, *struggling learners* refers to 1) students with identified disabilities that affect learning, particularly cognition-based disabilities (e.g., learning disabilities, attention-deficit/hyperactivity disorder [ADHD], mild-to-moderate developmental disabilities), and 2) students who experience significant difficulties with learning in school but are not identified as having a disability—traditionally identified as students at risk for school failure.

Students from diverse cultures and students who are English language learners may also struggle to learn mathematics (U.S. Department of Education, 2015). Although these groups are not the focus of this book, they may benefit from many of the strategies described. Language plays an important role in learning mathematics, and many of the strategies support students' use of both written and oral language. The emphasis on communication and classroom discourse can provide productive opportunities for English language learners to interact with English speakers and negotiate meaning as they develop their English language skills and conceptual understanding of mathematics (Baker et al., 2014).

FIVE COMPONENTS OF MEANINGFUL AND EFFECTIVE MATHEMATICS INSTRUCTION

To achieve success with the K–12 mathematics curriculum, struggling learners need teachers who understand why they have difficulty learning mathematics; instruction that addresses their unique learning needs, thereby allowing them to understand mathematics; and teachers who are committed to continuing to work with them until they reach understanding.

To facilitate and continually support such success among struggling learners, teachers must adopt and implement five components of meaningful and effective mathematics instruction. Figure 1.1 illustrates how the five components work together to support successful mathematics learning.

1. *Identify and Understand the Mathematics:* Teachers should explicitly identify, deeply understand, and teach the big ideas in mathematics. How a teacher

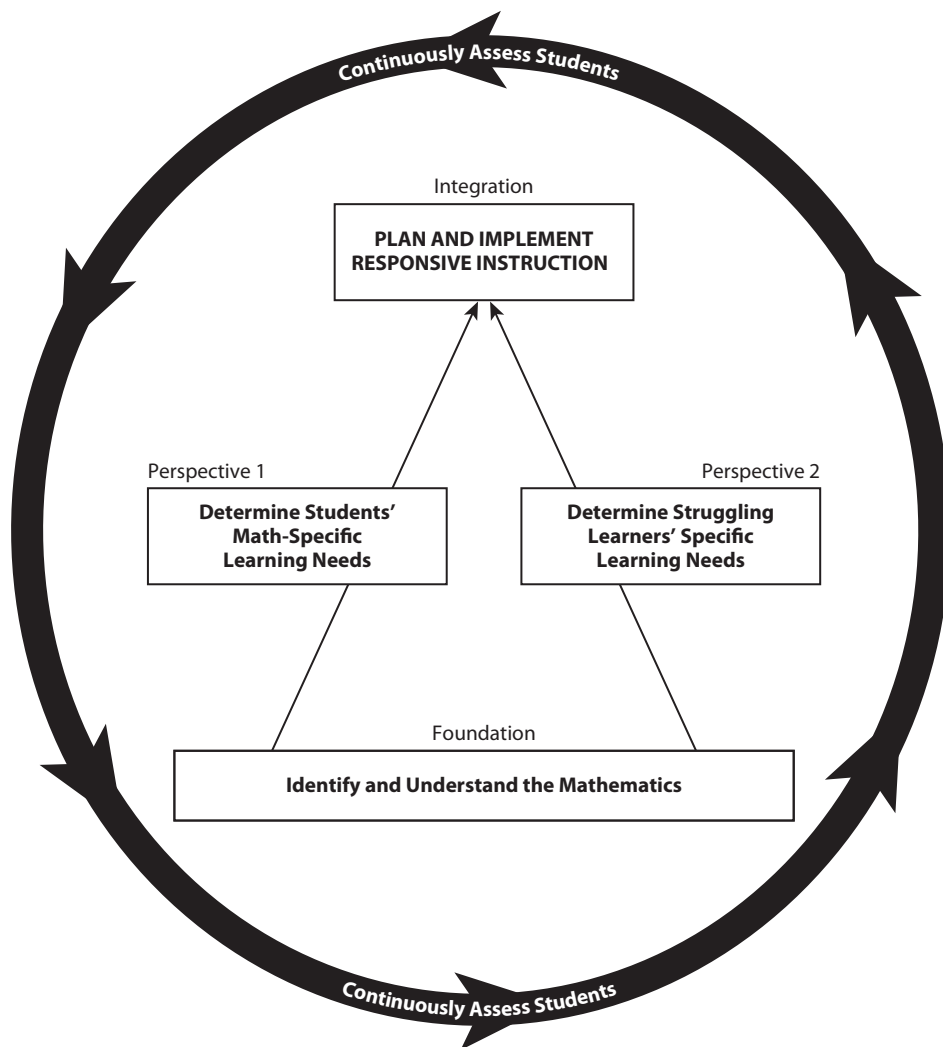


Figure 1.1. The Teaching Mathematics Meaningfully Process consists of five components of meaningful and effective mathematics instruction.

identifies and understands the mathematics lays the foundation for what is ultimately taught. Consequently, a deep understanding of the mathematics should serve as the foundation of every instructional choice you make.

2. *Continuously Assess Students:* Teachers should continuously assess student learning and progress and make informed instructional decisions based on these assessments. Purposeful and continuous assessment of student performance shows how well students understand target ideas and can do the target mathematics. Continuous assessment will help you determine and then respond to your students' learning needs in day-to-day instruction and long-term planning.

As an educator, you should think about your students from two crucial perspectives: determining their math-specific needs and determining specific learning needs of any struggling learners (see the next two components). Both are crucial to understanding your students' specific learning needs. Continuous assessment is an integral part of staying informed about your students' math-specific needs as well as any specific needs struggling learners might have.

3. *Determine Students' Math-Specific Learning Needs:* Teachers should determine where their students are in terms of their mathematical thinking, their knowledge and skills, and their gaps and misconceptions.
4. *Determine Struggling Learners' Specific Learning Needs:* Teachers should identify common performance traits of struggling learners, understand learning characteristics of struggling learners, and understand curriculum factors that create barriers to learning.

When they integrate these four components successfully, teachers are able to

5. *Plan and Implement Responsive Instruction:* Teachers make mathematics accessible through responsive instructional planning and teaching. They teach mathematics in a meaningful way.

The first component along with the third and fourth components must ultimately be integrated to teach math effectively and meaningfully. At the same time, the second component, continuous assessment, influences decisions teachers make throughout the entire process. Together, these components result in the fifth component: *Plan and Implement Responsive Instruction*.

Throughout this book, we use a figure that will “grow” to emphasize the importance of considering and integrating both math- and student-specific information when planning and implementing responsive mathematics instruction for students with disabilities and other struggling learners. We will add the components and supporting pieces of meaningful and effective mathematics instruction as you learn about them. Keep in mind that deep understanding of the mathematics is the foundation of your teaching and basis for all instructional decisions.

Identify and Understand the Mathematics means to identify and understand the mathematics standard, the relevant mathematical practice(s) for students, and the related learning trajectory. This math content knowledge must be accurately reflected in the assessment tasks you use to *Continuously Assess Students*.

After determining the mathematics content and beginning to think about appropriate assessments, you should *Determine Students' Math-Specific Learning Needs* as well as *Determine Struggling Learners' Specific Learning Needs* (see Figure 1.1); both educational perspectives further inform your instructional decisions. Determining students' math-specific learning needs requires careful analysis and, again, assessment based on your understanding of where students are on the learning trajectory for a given concept. You then use this understanding to determine target ideas for instruction. Determining the specific learning needs of your struggling learners also requires analysis and planning, based on students' observed performance traits, their characteristics as learners, and any potential barriers within the curriculum itself.

The conclusions you reach based on these analyses will guide you as you *Plan and Implement Responsive Instruction*—that is, instruction that is responsive to your students' needs. Doing so involves developing an instructional hypothesis you will use to tailor your instruction to their needs. After forming this hypothesis, planning instruction, and implementing it, you will use the results (including student performance data) to reflect and make decisions about how to improve instruction.

These five components of meaningful and effective mathematics instruction are not all-encompassing. However, they provide educators with an informed, multistep process for effectively teaching mathematics to struggling learners. Each component is an important part of effective math instruction for these learners; its importance is supported by literature in both special education and mathematics education. This chapter briefly introduces you to these critical components; subsequent chapters further develop each component.

Identify and Understand the Mathematics

To fully engage in responsive teaching, teachers must have a deep understanding of the big ideas in mathematics. Mathematics standards documents such as the following form a starting point for this understanding:

- *Principles and Standards for School Mathematics* (PSSM; National Council of Teachers of Mathematics [NCTM], 2000)
- *Common Core State Standards* (CCSS; National Governors Association [NGA] Center for Best Practices & Council of Chief State School Officers [CCSSO], 2010)

These documents provide grade-level expectations for what students should understand and be able to do in terms of content, as well as the multiple ways in which students should interact with mathematics.

Teaching for Deep Learning

Teaching the big ideas in mathematics for struggling learners has been a topic of discussion in the literature for decades (e.g., Carnine, Dixon, & Silbert, 1998; Cawley, Parmar, Yan, & Miller, 1998; NCTM, 2000; Parmar & Cawley, 1991). For example, Cawley and colleagues (1998) emphasized the importance of moving beyond basic skills instruction for struggling learners. They advocated the need for students to reason about the mathematics they do, in an effort to help students build connections between and among mathematical concepts. Carnine and colleagues (1998) promoted teaching big ideas that cut across the mathematics curriculum as a method for helping struggling learners apply the same idea in other mathematical areas (e.g., the use of area models for multiplication of whole numbers and then for multiplication of fractions, decimals, and polynomials). Baroody (1987) contended that traditional ways of teaching mathematics (i.e., a skills-only approach) do not meet struggling students' developmental or psychological needs, resulting in a lack of understanding and significant gaps in their mathematical knowledge. In essence, focusing on the big ideas instead of focusing solely on individual skills and concepts provides opportunities for students to construct connections across those various skills and concepts and acquire deep learning.

The “What” of Mathematics—Content

In the CCSS (NGA Center for Best Practices & CCSSO, 2010), the K–5 mathematics curriculum is organized according to content domains and big ideas related to content. These five domains can be thought of as the “what” of the mathematics curriculum:

- Operations and Algebraic Thinking
- Number and Operations in Base Ten
- Number and Operations—Fractions
- Measurement and Data
- Geometry

Each content domain has unique features and ideas that allow for its categorization as a separate area of mathematics (NGA Center for Best Practices & CCSSO, 2010). This separation is frequently carried to an extreme, though, so some people think of mathematics as separate bodies of ideas that have isolated rules and procedures that must be memorized (NCTM, 2014a; National Research Council, 2001). This simplified perspective is a narrow and false view of what mathematics really is about. It is true that mathematics is composed of different areas, such as algebra and geometry, but these areas are related in numerous and significant ways.

The “How” of Mathematics—Practices

This book also emphasizes the big ideas related to how students “do” mathematics because how students and teachers interact with the curriculum is intricately tied to what students ultimately learn and understand. The type of understanding and internalization of mathematics advocated by PSSM and now CCSS is best developed through instruction that engages students in the processes for doing mathematics (using mathematical understandings in meaningful ways to develop deeper, more connected mathematical knowledge), in contrast to instruction that emphasizes learning isolated mathematical concepts and skills.

PSSM (NCTM, 2000) identified five process standards (problem solving, communication, connections, representation, and reasoning) as ways students should interact with mathematics as they learn the content. The CCSS subsequently extended these five process standards to the Common Core Eight Standards for Mathematical Practice (NGA Center for Best Practices & CCSSO, 2010) to provide more specificity about how students need to engage with mathematics to improve their mathematical understanding, reasoning, and problem solving:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Chapter 2 provides more detailed information on these practice-oriented big ideas (how a learner engages with mathematics) as well as the content-oriented big ideas (the “what” of mathematics).

Continuously Assess Students

Continuous assessment of learning simply means that educators should evaluate what students know and can do before, during, and after instruction. Before teaching any mathematical concept or skill, teachers should evaluate students’ prerequisite knowledge and skill set as well as any experiences and interests they have that might relate to the target concept or skill. This snapshot of students’ existing knowledge and understanding provides insight into their level of competency with the related mathematical concepts and skills; their level of understanding (concrete, representational, or abstract); whether they can choose an example of a concept or skill (i.e., receptive understanding) or demonstrate their understanding without being provided choices (i.e., expressive understanding); and whether they have procedural knowledge, conceptual knowledge, or both.

Evaluating student understanding during instruction allows educators to monitor success so that changes can be made immediately. This prevents the loss of valuable instructional time and helps teachers to avoid surprises such as waiting until the class moves on to the next concept or unit before realizing—too late—that some students did not understand or develop more sophisticated reasoning.

After instruction, evaluating students’ understanding of the target concept provides teachers with a foundation for planning further instruction. In some cases, students might demonstrate sufficient understanding to move to the next target concept. In other cases, they may need additional instruction or response opportunities to become proficient (i.e., able to demonstrate understanding of the concept or perform the skill with a high level of accuracy and at a satisfactory rate). Determining this information after instruction occurs and before planning the next lesson ensures that teachers plan subsequent instruction that best meets students’ learning needs. Chapter 5 describes formative assessment practices that you can use to assess students in practical yet informative ways to guide your instructional decision making.

The literature has long advocated continuous assessment of students’ understanding to make informed instructional decisions in mathematics. A review by Black and Wiliam (1998a) of more than 250 studies strongly supported the notion that students’ learning is improved considerably when teachers consistently use formative assessment to guide their instruction. Assessment procedures such as curriculum-based assessment and curriculum-based measurement (CBM) result in teachers gaining greater awareness of their students’ learning needs and day-to-day progress in meeting learning goals and objectives (e.g., Allinder, Bolling, Oats, & Gagnon, 2000; Miller & Mercer, 1993; Shafer, 1998; Woodward & Howard, 1994). NCTM (1995, 2000, 2014a) has

long maintained that assessment should be an integral part of instruction, providing information about the students' learning not only to the teacher but also to the students themselves. With this information, teachers are better able to modify their instruction, and students are better able to modify their activity, all with the ultimate goal of students coming to a deeper understanding of the concepts being studied.

Determine Students' Math-Specific Learning Needs

From a mathematics education perspective, in order to effectively intervene with struggling learners, teachers must understand the evolution of students' reasoning about mathematical ideas. Mathematics has typically been taught from an adult's perspective, with very little, if any, consideration of how a child or novice understands mathematics (e.g., Baroody, Bajwa, & Eiland, 2009; Carpenter, Fennema, Franke, Levi, & Empson, 2015; Wright, Ellemor-Collins, & Tabor, 2012). In this book, we capitalize on the research that has resulted in learning trajectories, which are detailed descriptions of the evolution of children's mathematical reasoning and learning about specific mathematical ideas over long periods of time.

As an example of a learning trajectory, consider the following task that is similar to ones typically posed in the earlier grades:

Ben has 7 carrots. His friend gives him 4 more. How many carrots does Ben have now?

Teachers might encourage children to "count on" from 7 to find the answer. However, before children can count on from a given number, they must conceive of the given number, 7, as a composite unit. First, children must count (by ones) to the given number to give meaning to that amount. So, for this task, a child may have to count by ones to 7 and then keep track of 4 more counts on his or her fingers to reach the total of 11. Counting to 7 and then counting 4 more may seem inefficient to an adult, but it is an appropriate strategy for many children. Another child may be able to start from 7 and count on 4 more to get to 11. Yet another child may be able to use his or her understanding that 7 is 3 away from 10, and then one more is 11. These different strategies reflect different positions on a learning trajectory that details how children understand number, counting, and the operation of addition (Wright, Stranger, Stafford, & Martland, 2009).

The perspective provided by learning trajectories contrasts with the notion that when learning a particular mathematical idea, there is only one way to understand it. Learning trajectories help teachers better understand their students because they provide a road map of students' thinking that ranges from initial, intuitive ideas to more complex and formal understandings. Learning trajectories also help teachers recognize their students' current level of understanding of the target mathematical concepts and skills from the CCSS or other state standards and guide them in their efforts to build on that thinking during subsequent instruction.

The field of mathematics education has long advocated for developmentally appropriate instruction, through which teachers provide mathematics instruction that is consistent with students' developing cognitive abilities (Carpenter et al., 2015; Clements & Sarama, 2009; Kami, 2000; Steffe & Olive, 2010). Without the

knowledge of how children learn mathematics, educators tend to teach in ways that can hinder meaningful learning (e.g., Kami, 2000). Determining the knowledge, skills, and common misconceptions related to the associated math standards is an essential element of tailoring your instruction to meet struggling learners' math-specific needs. Tapping into learning trajectories related to associated math standards can support your efforts in tailoring instruction. Chapter 3 provides more detail about learning trajectories.

Determine Struggling Learners' Specific Learning Needs

From a special education perspective, to effectively support struggling learners, teachers must understand the common math performance traits they exhibit, the barriers they face when learning mathematics, and the causes of these barriers. In Chapter 4, we describe a set of common performance traits that indicate math learning difficulties and focus on two major barriers: barriers that result from the learning characteristics of struggling learners and barriers that result from how these learning characteristics interact with the mathematics curriculum and how it is taught.

The following math performance traits indicate learning difficulties:

- Demonstrating knowledge and skill for some mathematical domains and not others, or for certain standards within a domain and not others
- Demonstrating faulty mathematical thinking or ineffective strategies when problem solving
- Being able to compute or engage in problem solving accurately but at a very slow pace
- Having difficulty with generalizing knowledge and skills to other mathematical concepts, skills, and contexts
- Demonstrating mathematical abilities at one point in time but then being unable to demonstrate the same abilities later
- Avoiding engagement in certain mathematical tasks

The following barriers can result from the learning characteristics of struggling learners:

- Learned helplessness (i.e., because of repeated failure, students believe they have no control over their learning, so they stop trying)
- Passive approaches to learning; impairments in processing and metacognition (i.e., thinking about one's thinking)
- Gaps in knowledge, understanding, or skills
- Math anxiety
- Attention and memory difficulties
- Difficulties with metacognition
- Processing disabilities
- Reading disabilities

The following curriculum-related factors can create barriers to learning:

- The degree to which effective mathematics practices for struggling learners are utilized across instructional tiers within multi-tiered systems of supports (MTSS)
- Instructional pacing
- The extent to which mathematics instruction emphasizes the integration of conceptual understanding with procedural fluency (i.e., the ability to apply procedures accurately, efficiently, and flexibly)
- The level of emphasis placed on teaching foundational number and operations and algebraic thinking concepts or skills within MTSS
- The varying nature of particular mathematics content

Teachers who understand how these sets of factors can affect mathematical learning are better equipped to understand the needs of struggling learners generally and individually, which in turn provides them with a better-informed perspective for planning and implementing responsive mathematics instruction.

Plan and Implement Responsive Instruction

The primary purpose of this book is to help educators make mathematics accessible to struggling learners. The key to this access is responsive instruction, which is simply instruction tailored to the students' needs. Educators must be creative in thinking about how to engage struggling learners in meaningful mathematics learning experiences through planning and implementing methods, practices, and procedures that directly address these students' learning characteristics and needs. When students can process a concept in ways that make sense to them or that are accessible given their own learning abilities and needs, they are more likely to understand the concept. To be responsive, instruction must be continuously informed and revised by assessment results and students' learning needs, mathematical and otherwise.

The literature that informs effective instructional practices for struggling learners builds on three of the critical components of meaningful and effective mathematics instruction: *Identify and Understand the Mathematics*, *Continuously Assess Students*, and *Determine Students' Math-Specific Learning Needs*. As the research on struggling learners in mathematics has evolved, there has been greater awareness that mathematics instruction should be more than direct teaching of basic skills and that struggling learners can learn mathematics at much deeper levels of understanding than previously believed (Jayanthi, Gersten, & Baker, 2008). This is especially true when instructional practices using different levels of teacher support are implemented (Mercer, Lane, Jordan, Allsopp, & Eisele, 1996). Teachers can make instructional choices about the kind and degree of support they provide students; they most successfully implement this continuum of instructional choices when they consciously and purposefully consider *what* struggling learners are ready to learn, how they will be asked to *do* it, how it relates to their *knowledge*, and the *barriers* that might make learning difficult for them. In this way, their teaching is responsive to students' needs. When such responsive teaching is applied to struggling learners, students are more likely to experience success.

A growing body of research documents mathematics instructional practices that respond to struggling learners' needs (e.g., Baxter, Woodward, & Olson, 2005; Bottge, Heinrichs, Mehta, & Hung, 2002; Cawley et al., 1998; Gersten, Beckman, et al., 2009; Gersten, Chard, et al., 2009; Jayanthi et al., 2008; Kroesbergen & Van Luit, 2002, 2003; Lock, 1996; Maccini & Gagnon, 2000; Mercer, Jordan, & Miller, 1996; Miller, Butler, & Lee, 1998; National Mathematics Advisory Panel, 2008; National Research Council, 2001; Newman-Gonchar, Clark, & Gersten, 2009; Owen & Fuchs, 2002; Vaughn, Gersten, & Chard, 2000; Woodward & Brown, 2006). Chapters 6–11 provide detailed information about effective mathematics instruction for struggling learners, including 1) the importance of approaching it flexibly without a one-size-fits-all perspective; 2) a contextual summary of research; 3) descriptions of 11 Essential Instructional Approaches (EIAs) for mathematics based on the relevant research and literature; 4) suggestions for utilizing the EIAs to provide access and equity for struggling learners; and 5) suggestions for improving mathematics instruction within MTSS.

ACCESS AND EQUITY IN MATHEMATICS EDUCATION THROUGH SYSTEMATIC INSTRUCTIONAL DECISION MAKING

NCTM's (2014a) position statement on access and equity in mathematics education calls for a firm commitment from educators and other stakeholders to embrace the expectation that all students can learn and become mathematically proficient through access to high-quality curriculum and instruction. The phrase *all students* includes the struggling learners identified previously in this chapter. The critical components for effective mathematics instruction for struggling learners, introduced in this chapter and developed throughout the book, can help you realize NCTM's expectation as you conceptualize, plan, and evaluate your own mathematics instruction. The final chapter of the book provides an in-depth description of the Teaching Mathematics Meaningfully Process, which integrates all five components of meaningful and effective mathematics instruction. Appendix E includes a case study that illustrates how teachers can utilize this decision-making process to make responsive instructional decisions for struggling learners, thereby making mathematics more accessible to and equitable for them.

SUMMARY

Components of Teaching Mathematics Meaningfully

In this chapter, we introduced five components of meaningful and effective mathematics instruction:

1. Identify and Understand the Mathematics
2. Continuously Assess Students
3. Determine Students' Math-Specific Learning Needs
4. Determine Struggling Learners' Specific Learning Needs
5. Plan and Implement Responsive Instruction

Although these aspects of making mathematics meaningful are not all-encompassing, they provide educators with an informed, multistep process for effectively teaching mathematics to struggling learners that is supported by literature in both special education and mathematics education.

X TAKE ACTION

In this book's preface, we emphasized that no matter where you are in your professional journey—prospective, new, or veteran teacher—there is always more to discover about teaching and learning. Being able to critically examine your practice for areas of strength and areas for improvement is pivotal for professional growth.

Activity 1.1: Self-Observation: Strengths and Opportunities for Improvement

At the beginning of this chapter, we introduced five components of meaningful and effective mathematics instruction. Take a minute to reflect on your current strengths and opportunities for growth in these five areas. Take Action Activity 1.1 in Appendix A provides a blank photocopiable form that allows you to record your strengths and opportunities for growth (see also the About the Activities and Forms page in the front of this book for information on how to download a printable copy).

We carefully crafted this book to support your growth in each area, and this growth will only be enhanced by your candid articulation of what you already know and what you hope to get out of this book in each area. This is your first opportunity to engage in the pivotal activity of examining your practice as you read; however, we hope that you will continue to revisit and revise this form as you successfully incorporate these five components into your own practice.

I

Identify and Understand the Mathematics

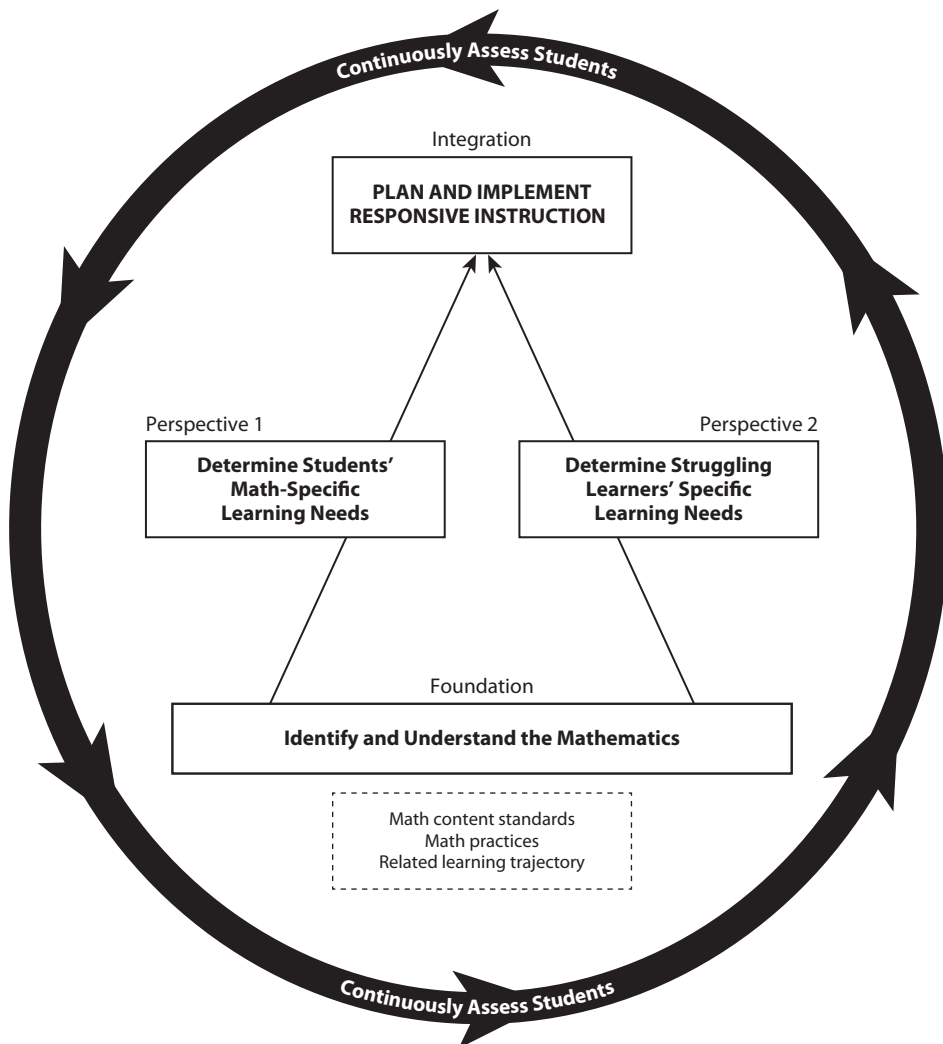


Figure I.1. The *Identify and Understand the Mathematics* component of the Teaching Mathematics Meaningfully Process.